

Constraining scenarios of the soft/hard transition for the pion electromagnetic form factor with expected data of 12-GeV Jefferson Lab experiments and of the Electron–Ion Collider

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Abstract

It has been shown previously [1] that a non-perturbative relativistic constituent-quark model for the π -meson electromagnetic form factor allows for a quantitative description of the soft/hard transition, resulting in the correct Quantum-Chromodynamical asymptotics, including normalization, from the low-energy data without further parameter tuning. This happens universally whenever the constituent-quark mass is switched off. The energy range where the transition happens is therefore determined by the quark-mass running at intermediate energies and is not tightly constrained theoretically. Here we consider possible ways to pin down this energy range with coming experimental data. We demonstrate that expected experimental uncertainties of the 12-GeV Jefferson-Lab data are larger than the span of predictions of the model, so these data might be used for testing the model but not for determination of the soft/hard transition scale. Contrary, the projected Electron-Ion Collider will be capable of pinning down the scale.

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I. INTRODUCTION

Making connections between high-energy (hard) and low-energy (soft) models of hadrons, that is between the Quantum Chromodynamics (QCD) and effective theories of strong interactions working in the infrared limit, is one of the major challenges of modern particle theory. The electromagnetic form factor of the charged π meson, F_π , represents a particularly interesting observable in this context. On one hand, its high-energy asymptotics is well defined within the perturbative QCD [2–4]. On the other hand, precise experimental data in the soft region allow to directly trace the evolution of the observable with the momentum transfer, Q^2 , up to $Q^2 \gtrsim 2 \text{ GeV}^2$, that is close to the range where one expects the hard behavior to start settling down.

The F_π form factor is remarkable in one more aspect which we will exploit here. It is probably the only observable for which a successful low-energy theory exists which gives the correct QCD asymptotics quantitatively without a dedicated parameter tuning [1]. The soft/hard transition is governed by switching the constituent-quark mass M off in the model, and wherever the mass is switched off, the QCD asymptotics settles down, provided the low-energy description works well. Therefore, no additional parameters are required to be tuned to reproduce quantitatively the QCD asymptotics. The model uses the $M(Q^2)$ dependence as an input, and various ways of switching the mass off are allowed, just because the asymptotics is universal and is determined by the infrared, and not intermediate-scale, parameters. Therefore, the model, in its present form, does not predict the energy scale at which the soft/hard transition, that is switching $M(Q^2)$ off, takes place; this scale is to be determined either by a detailed model for $M(Q^2)$ or experimentally. Unfortunately, detailed models for the M running available in the literature contain a number of free parameters and can hardly be used for this purpose. Here, we explore prospects for experimental determination of this scale from future F_π measurements.

The rest of the paper is organized as follows. In Sec. II, we briefly review the model for the pion form factor which gives the correct QCD asymptotics starting from the low-energy physics. In Sec. III, we describe how the unknown ingredient of the model, the dependence of the effective constituent-quark mass on Q^2 , is constrained. We proceed in Sec. IV with estimates of the impact future data may have on these constraints, working within two representative scenarios; then we briefly conclude in Sec. V.

II. THE MODEL FRAMEWORKS

We adopt a well-elaborated model for the pion form factor [1, 5–9] originally developed as a low-energy theory based on the Poincaré invariant constituent-quark model. It exploits the instant form of Relativistic Hamiltonian Dynamics (see e.g. Ref. [10]). The use of the Modified Impulse Approximation [6] provides for the full relativistic invariance and eliminates certain drawbacks of the original instant form. The pion form factor, $F_\pi(Q^2)$, is given by rather cumbersome but explicit expressions [6] collected in Ref. [1], which we do not quote here. The low-energy model has two free parameters, the constituent-quark mass M and the wave-function confinement scale b (the actual choice of the wave function does not have any significant effect on the result, see Refs. [5, 8]). These two parameters are tuned to reproduce correct experimental values of the pion decay constant f_π and of the pion charge radius. It is remarkable that these parameters were fixed from the low-energy data (actually in 1998, Ref. [5], by making use of the $F_\pi(Q^2)$ measurements at $Q^2 \lesssim 0.26 \text{ GeV}^2$ [11]), so no room to tune them remained. The predictions of Ref. [5] have been subsequently verified by new measurements of $F_\pi(Q^2)$ up to $Q^2 \simeq 2.5 \text{ GeV}^2$, an order of magnitude higher, and are in excellent agreement with all present-day data.

In parallel with this phenomenological success, the model has an interesting, if not miraculous, theoretical advantage. It has been noted in Ref. [12] that the asymptotical behavior $Q^2 F_\pi(Q^2) \sim \text{const}$, predicted by QCD [13, 14], is obtained in this model at $M \rightarrow 0$. This is however not a full story: introducing explicit $M(Q^2)$ dependence in such a way that $M(0)$ is taken from the original model but $M(\infty) = 0$, we obtained in Ref. [1] the numerical coefficient of this asymptotics which appeared to reproduce the QCD predictions quantitatively, without any parameter tuning, for every possible way of switching the quark mass off. The QCD asymptotics [2–4] is

$$Q^2 F_\pi(Q^2) \simeq 8\pi\alpha_s^{(1)}(Q^2)f_\pi^2, \quad (1)$$

where $\alpha_s^{(1)}(Q^2)$ is the one-loop QCD coupling constant (extension to higher loops is not straightforward, see e.g. Refs. [15, 16]). The right-hand side of Eq. (1) is determined by two parameters, f_π (determined in the low-energy theory) and the QCD scale Λ_{QCD} (which is related, though not explicitly, to the low-energy confinement parameter b). The fact that, by fixing f_π and the charge radius in the low-energy theory, we immediately reproduce Eq. (1) quantitatively, is an important advantage of the model, not seen in other approaches.

The idea of switching the constituent-quark mass M off in order to obtain the form-factor behaviour at high Q^2 was put forward in Ref. [17] in the frameworks of the light-front approach [18]; however, Eq. (1) was not obtained there. In Ref. [1], we used a parametrization for $M(Q^2)$ inspired by Ref. [17] but corrected for effects of a one-gluon exchange,

$$M(Q^2) = M(\infty) + (M(0) - M(\infty)) \frac{1 + e^{-\mu^2/\lambda^2}}{1 + e^{(Q^2 - \mu^2)/\lambda^2}} L(Q^2), \quad (2)$$

$$L(Q^2) = \frac{1}{1 + \log \frac{Q^2 + \mu^2}{\mu^2}}. \quad (3)$$

The soft/hard transition is therefore governed by two parameters, μ and λ , of the $M(Q^2)$ function. The μ and λ parameters determine the position and the steepness of the transition between $M(0)$ and $M(\infty)$. The boundary value $M(\infty) \rightarrow 0$ while $M(0) = 0.22$ GeV is fixed, like in previous studies, to reproduce f_π and the pion charge radius correctly. We note that the parametrization (2), (3) describes well the $M(Q^2)$ dependence obtained, within certain assumptions about free parameters, in complicated non-perturbative dynamical models [19, 20], see Ref. [1] for details and illustrations.

To summarize this section, we have a predictive quantitative model for the pion form factor whose low-energy parameters are fixed and determine the correct high-energy asymptotics automatically, but the soft/hard transition is parametrized by $M(Q^2)$, that is by two parameters (μ, λ) . We turn now to constraining these parameters.

III. CONSTRAINING $M(Q^2)$

We start with theoretical constraints which are determined by the limits of applicability of the model. While, technically, Eq. (2) implies that M is always decreasing for arbitrary μ and λ , it appears that it cannot decrease too slow. Indeed, for each fixed Q^2 , there exists a value $M_{\max}(Q^2)$ beyond which the form factor, considered as a function of M , ceases to be monotonic. This, in turn, implies that the form factor as a function of Q^2 may cease to be monotonically decreasing. We calculate this $M_{\max}(Q^2)$ numerically and determine, from the requirement $M(Q^2) < M_{\max}(Q^2)$, the corresponding restriction on the parameters (μ, λ) . This bound (“the consistency limit”), applicable in any case, is presented in Fig. 1 as a long-dashed line: the allowed region is to the left of this line. Note that the requirement of perturbativity at large Q^2 we used in Ref. [1] is qualitatively similar to this bound.

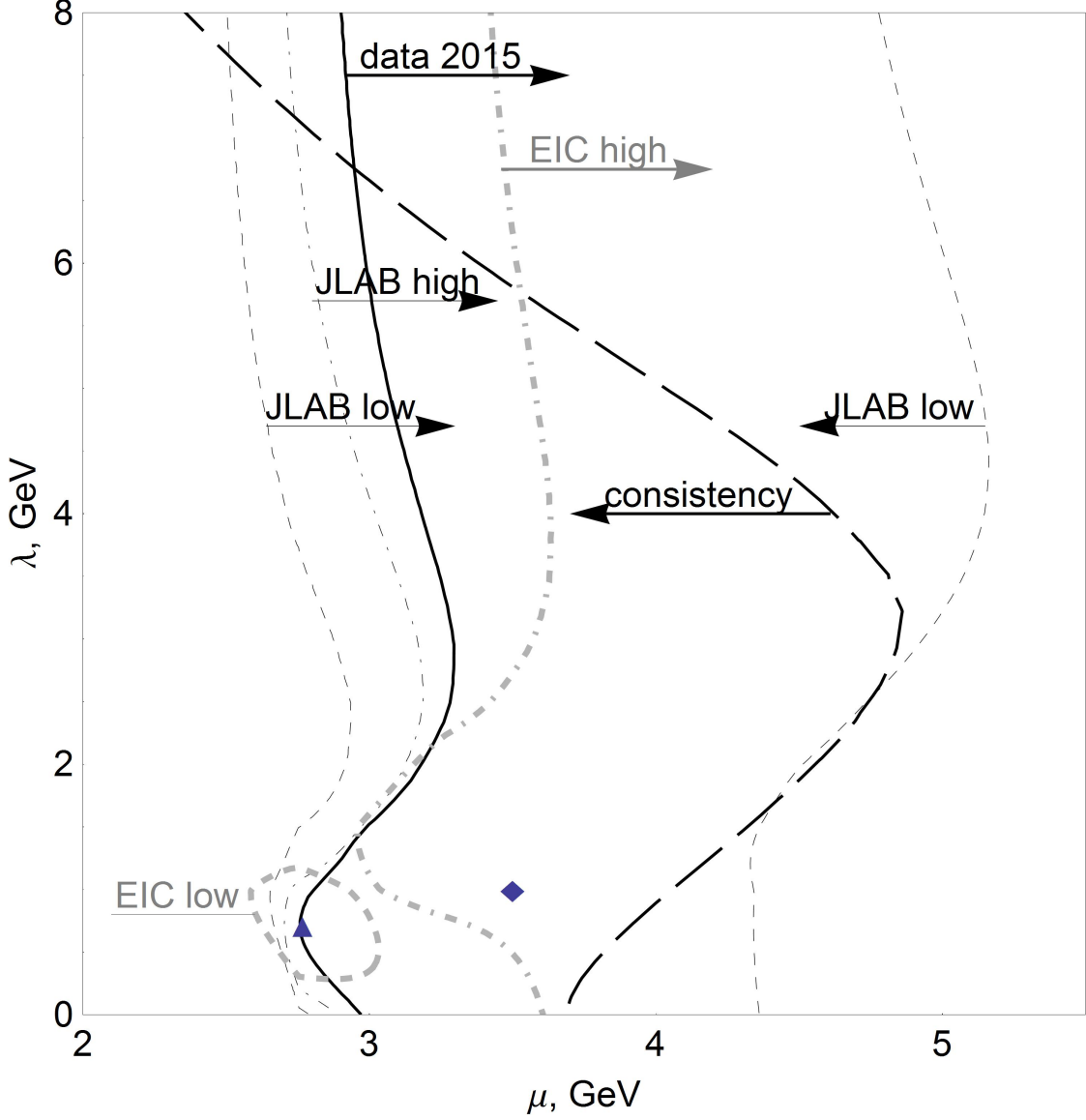


FIG. 1. Allowed regions of the plane of parameters μ and λ which govern the quark-mass evolution, see text. Long-dashed line: the consistency limit (region to the left of the line is allowed; relevant for all cases). Full black line: the 95% CL bound from the present data (region to the right of the line is allowed). Other lines: 95% CL example bounds from future data for the “low-scale” (dashed; the allowed region is bound by the lines) and “high-scale” (dot-dashed; the allowed region is to the right from the lines) scenarios of the soft/hard transition; thin lines assume 12-GeV JLab data, thick gray lines assume EIC data. The values of μ and λ assumed for the “low-scale” and “high-scale” scenarios are shown by the triangle and the diamond, respectively.

The other kind of constraints come from experimental measurements of F_π at relatively high Q^2 . As we have already pointed out, the original model with $M = \text{const}$ predicted the form-factor values up to $Q^2 \sim 2.6 \text{ GeV}^2$ with high accuracy, hence an early departure from the constant-mass scenario might result in a disagreement with data. The corresponding bound on $M(Q^2)$ is obtained from the requirement of agreement, at the 95% confidence level (CL), of the corresponding $F_\pi(Q^2)$ function with the data points, tested by means of the usual chi-square method. For demonstration purposes, we also define the “soft/hard transition scale” Q_{trans}^2 as the value of Q^2 at which the difference between the predicted $Q^2 F_\pi(Q^2)$ and Eq. (1) is one half of its maximal value, that is the form factor is half way from its nonperturbative values to the QCD asymptotics.

By making use of all present-day data described in Ref. [21], we obtain constraints on $M(Q^2)$ which are presented in Fig. 1 in terms of μ and λ (the full line; the region to the right of the line is allowed). The corresponding range of allowed $F_\pi(Q^2)$ is shown as a gray band in Fig. 2 (see Ref. [1] for more plots). In terms of the soft/hard transition scale, this constraint is $Q_{\text{trans}}^2 > 8.5 \text{ GeV}^2$.

Having determined the constraints on $M(Q^2)$ from the present data, we are ready to discuss prospective bounds from future experiments.

IV. EXPECTED CONSTRAINTS FROM FUTURE DATA

Experimental prospects of the measurements of the pion form factor are briefly summarized in Ref. [24]. They include the approved E12-06-101 experiment at the upgraded Jefferson Laboratory (12-GeV JLab) facility and measurements at the projected Electron-Ion Collider (EIC). More details may be found in the experimental proposal [25] for the 12-GeV JLab and in the talk [26] for EIC. Hereafter, we will use the information about the Q^2 reach and projected error bars of F_π measurements at these facilities given in Refs. [25, 26] and reproduced in Ref. [24] (for EIC, we assume the energy of the ion beam of 5 GeV, the lowest one considered there). With the 12 GeV energy, JLab will be able to measure F_π for the momentum transfers up to $\sim 6 \text{ GeV}^2$ with the accuracy of $\sim 4\%$. For EIC, there exist various proposals under consideration; for the 5 GeV proton energy, F_π might be measured up to $Q^2 \sim 15 \text{ GeV}^2$ with the accuracy of $\sim 10\%$.

To proceed further, we restrict ourselves to two particular representative scenarios cor-

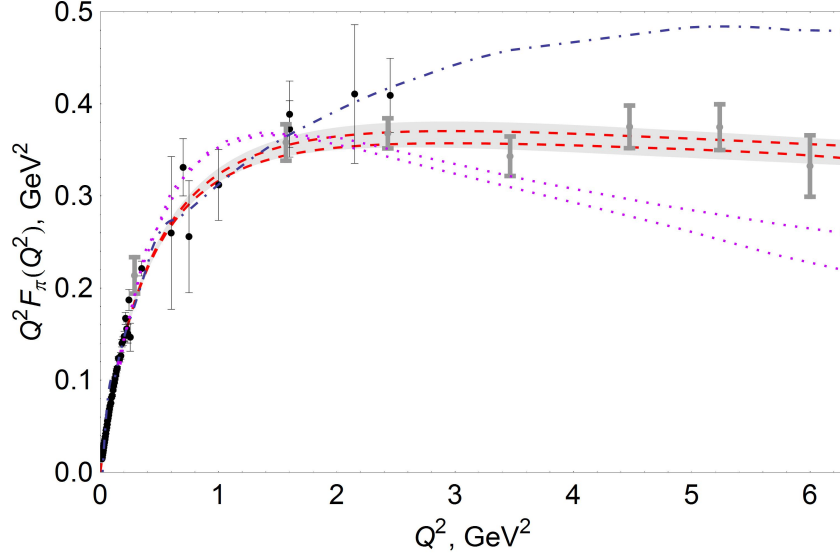


FIG. 2. The range of predictions of our model for the charged pion form factor allowed by the present data and model consistency for various scenarios of quark-mass running (gray band) together with predictions of Refs. [22] (area between two dotted lines) and [23] (dash-dotted line). Two dashed lines represent two particular scenarios for $M(Q^2)$ corresponding to the “low-scale” and “high-scale” soft/hard transitions discussed in the text. Existing experimental data points (see Ref. [1] for their description and list of references) are shown by black dots with thin error bars. A typical simulated example realization of expected 12-GeV JLab data, corresponding to the “high-scale” scenario, is shown by gray dots with thick gray error bars. It is evident that the JLab data would not help to choose between the allowed scenarios of quark-mass running within our model but would provide a good test of the model versus others.

responding to the “low-scale” and “high-scale” soft/hard transition. The (μ, λ) parameters of these scenarios are shown in Fig. 1 by symbols. The low-scale scenario corresponds to the lowest transition scale which agrees, at the 95% CL, with the present data ($\mu = 2.79$; $\lambda = 0.715$). The high-scale one is a typical representative point inside the allowed region ($\mu = 3.5$; $\lambda = 1.0$). The corresponding $F_\pi(Q^2)$ functions are shown in Figs. 2, 3 by dashed lines.

Having assumed particular values for μ and λ , and therefore a particular $F_\pi(Q^2)$ model curve, we simulate fake data points for a given experiment, scattered around the theoretical curve with the Gaussian distribution. The width of the distribution is determined by the error bars quoted in Refs. [25, 26]; the values of Q^2 for these fake “measurements” are also

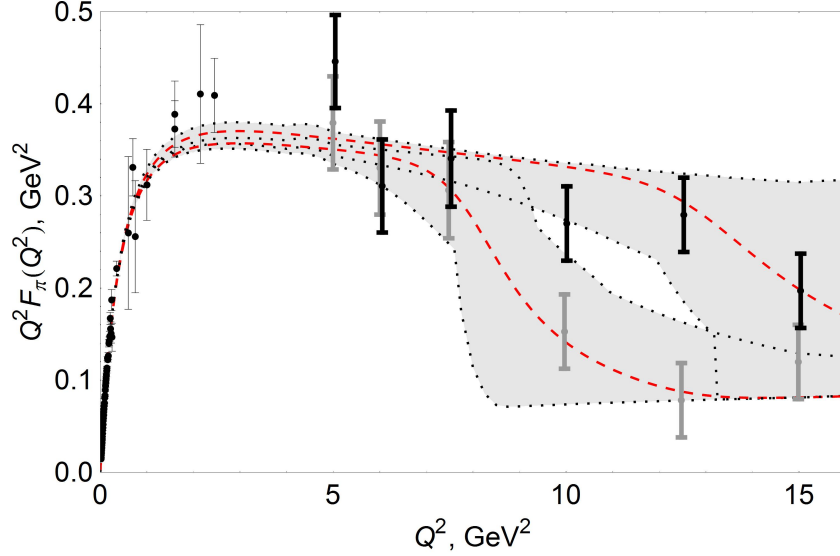


FIG. 3. The range of predictions of our model for the charged pion form factor allowed by simulated EIC data and model consistency for two particular scenarios for $M(Q^2)$ (dashed lines) corresponding to the “low-scale” and “high-scale” soft/hard transitions discussed in the text (gray bands bound by dotted lines). Existing experimental data points (see Ref. [1] for their description and list of references) are shown by black dots with thin error bars. Two typical simulated example realizations of expected EIC data, corresponding to the “high-scale” (black) and “low-scale” (gray) scenarios, are shown by dots with thick error bars. JLab 12-GeV simulated data are not shown for clarity. It is evident that the EIC data would make it possible to choose between the allowed scenarios of quark-mass running within our model.

taken from there. Then, these fake data are processed in the joint chi-square fit with the existing data. The results are presented in Figs. 1, 2, 3.

For 12-GeV JLab, one may see from Fig. 2 that the expected error bars of the F_π measurements exceed the width of the region allowed by the present data for our model. Therefore, these data are not expected to contribute much into the determination of the soft/hard transition scale, as illustrated in Fig. 1 in terms of (μ, λ) . However, we point out that the 12-GeV JLab data will be of crucial importance for testing the model itself. In Fig. 2, predictions of two alternative scenarios describing soft/hard transitions are also shown (see Ref. [1] for a more detailed discussion). Clearly, the precision of the expected 12-GeV JLab data would be sufficient to confirm or exclude the model we use here.

Contrary, the 5-GeV EIC data, though having large expected error bars, will be able to

disentangle the low-scale and high-scale transition scenarios within our approach, see Figs. 1, 3. The 95% CL constraints obtained in the two example scenarios are clearly separated. There still remains a formal degeneracy for $Q_{\text{trans}}^2 \gtrsim 16 \text{ GeV}^2$, but this range of momentum transfer, with the energy scale of order the b -quark mass, most probably corresponds to the perturbative QCD regime anyway.

V. CONCLUSIONS

We considered the effect of future experimental data on our understanding of the dependence of the pion electromagnetic form factor, F_π , on the momentum transfer squared, Q^2 , paying a special attention to the “soft/hard” transition region where the QCD asymptotics should settle down. We took advantage of a particular low-energy model which describes excellently the existing data and predicts the QCD asymptotics automatically, without parameter tuning.

Given the estimated precision of 12-GeV JLab and of EIC, as well as the expected range of the momentum transfer accessible to the instruments, we conclude that the coming JLab data may confirm or exclude our model while the EIC measurements would be able to pin down the soft/hard transition scale.

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